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International Journal of Heat and Mass Transfer 46 (2003) 181–183

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

On the treatment of open boundary condition for radiative transfer equation

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Received 10 May 2002

1. Introduction

The development of numerical techniques to solve the chemical reactive flows has received increased attention since the 1980s. Numerical computations of the differential equations related to reactive flows are defined in terms of boundary conditions. Implementing physically reasonable and sufficiently consistent boundary conditions requires a strong understanding of the interior and exterior phenomena. There are different domain of dependence and the zone of influence between radiative transfer equation and momentum and energy equations, and the treatment of boundary conditions for radiative transfer equation is not similar to that for momentum and energy equations.

Recently, the open boundary of computational domain is often taken as a black wall with local medium temperature in numerical simulation of radiative transfer [1–3]. The assumption of black wall is not consistent with the physical condition of open boundary. The objective of this article is to analyze the errors resulting from taking an open boundary as a black wall in numerical simulation of radiative transfer. A three-dimensional simplified rectangular furnace is taken as an example, and the discrete ordinates method is used to solve the radiative transfer equation in the furnace. By comparing the numerical results of radiative heat flux distribution for different computational domains with different open boundary locations, the conditions for treating the open boundary as a black wall with local temperature are studied.

2. Physical model

As shown in Fig. 1, we consider a three-dimensional simplified rectangular furnace filled with homogeneous absorbing, emitting, scattering and gray medium. The optical thicknesses of the furnace in x, y and z coordinates are $\tau_{xL} = 1.0$, $\tau_{yL} = 1.0$ and $\tau_{zL} = 5.0$, respectively. The scattering albedo of medium is $\omega = 0.5$ and the scattering phase function is $\Phi = 1 + \cos\theta$. The boundary at $\tau_z = 5.0$ is open, and the others are black walls. The temperature of the furnace is assumed to be one-dimensional and only dependent on z coordinate.

To analyze the errors resulting from taking the open boundary of computational domain as a black wall with local temperature, we consider three different temperature distributions as following:

$$T(\tau_z) = 3000 - 540\tau_z \tag{1}$$

$$T(\tau_z) = \begin{cases} 3500 - 3000\tau_z/7, & 0 \leqslant \tau_z < 3.5 \\ 2000, & 3.5 \leqslant \tau_z < 4 \\ 2000 - 1700(\tau_z - 4), & 4 \leqslant \tau_z \leqslant 5 \end{cases} \tag{2}$$

$$T(\tau_z) = \begin{cases} 3500 - 500\tau_z, & 0 \leqslant \tau_z < 3 \\ 2000, & 3 \leqslant \tau_z < 4 \\ 2000 - 1700(\tau_z - 4), & 4 \leqslant \tau_z \leqslant 5 \end{cases} \tag{3}$$

In all of the three temperature distributions, the temperature of furnace at the open boundary of $\tau_z = 5.0$ is 300 K and the surrounding is taken as a blackbody with 300 K. In actual engineering calculation, the computational domain is often taken as a part of the furnace and the outlet boundary locates in high-temperature zone. In following analysis, we take the numerical simulation results of total furnace volume ($\tau_{xL} \times \tau_{yL} \times \tau_{zL}$) as a comparing standard to analyze the errors resulting from setting the open boundary of computational domain in high-temperature zone and treating it as a black wall.

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PII: S0017-9310(02)00254-5

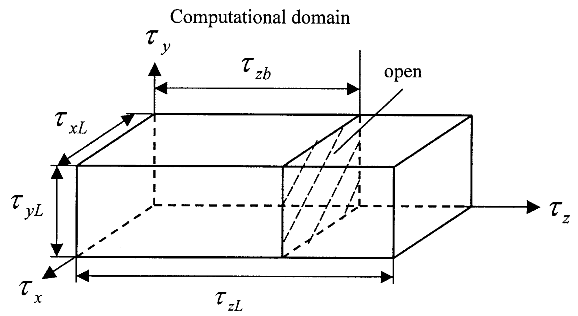

Fig. 1. Geometry of the simplified furnace.

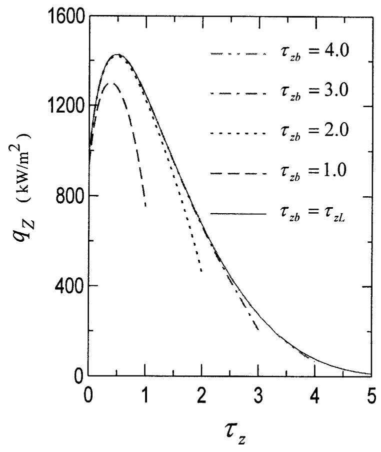

Fig. 2. The distribution of averaged axial net radiative heat flux for five different locations of the open boundary of computational domain in the case of temperature distribution given by Eq. (1).

3. Results and discussion

The discrete ordinates method [4,5] has emerged as a successful candidate which satisfies the requirements of compatibility with the numerical schemes used to model the other transfer modes and acceptable computational economy. In this paper, the S_6 discrete ordinates method with the level symmetric even quadrature [6] is used to solve the radiative transfer problem in the rectangular computational domain of $\tau_{xL} \times \tau_{yL} \times \tau_{zb}$ as shown in Fig. 1, in which τ_{zb} is the open boundary location of the computational domain and $0 < \tau_{zb} \leqslant \tau_{zL}$. For the sake of comparison, the averaged axial net radiative heat flux is defined as

$$q_z = \frac{1}{\tau_{xL}\tau_{yL}} \int_0^{\tau_{yL}} \int_0^{\tau_{xL}} \int_0^{4\pi} I\mu \,\mathrm{d}\Omega \,\mathrm{d}\tau_x \,\mathrm{d}\tau_y \tag{4}$$

where I is radiative intensity, μ is the direction cosines of travel direction of radiative ray in τ_z coordinate, and Ω is solid angle. The solution to the discrete ordinate equations of radiative transfer equation must be obtained iteratively. Once the radiative intensity is known, the averaged axial net radiative heat flux is determined approximately by numerical integration from Eq. (4). The detailed procedure of solution of discrete ordinate equation can be seen in Ref. [7], and will not be repeated here. Grid refinement was performed for the physical model to ensure that the essential physics are independent of grid size. The convergence of computations is declared as

$$\max\left\{\left|\frac{I^{\text{new}} - I^{\text{old}}}{I^{\text{new}}}\right|\right\} < 10^{-6} \tag{5}$$

where superscripts 'new' and 'old' denote the present and the previous iteration values, respectively.

Fig. 2 shows the distribution of averaged axial net radiative heat flux for five different locations of the open boundary in the case of temperature distribution given by Eq. (1). The gradient of axial temperature, $|\partial T/\partial \tau_z|$, is 540. As shown in Fig. 2, the errors resulting from taking the open boundary of computational domain as a black wall with local temperature increases with the open boundary of computational domain moving into high-temperature zone. When the open boundary of computational domain locates in the low-temperature zone, for example, $\tau_{zb} = 4.0$, the error can be omitted. This is because the thermal emission of low-temperature media near the open boundary is very small compared with the thermal emission of high-temperature media within computational domain. When the open boundary of computational domain locates in the high-temperature zone, for example, $\tau_{zb} = 1.0$, the error becomes large enough and cannot be omitted. The radiation emitted from the media at $\tau_{zb} = 3.0$ is attenuated significantly when it reaches at the site $\tau_z = 2.0$, as shown in Fig. 2, the open boundary can be set up in the site of $\tau_{zb} = 3.0$ and the computational errors in the interesting zone of $0 \leqslant \tau_z \leqslant 2$ are small. This means that, in order to get the reasonable numerical results, the open boundary needs to be set up far from the interesting zone more than 1.0 optical thickness if the open boundary of computational domain is treated as a black wall with local temperature.

Fig. 3 shows the distribution of averaged axial net radiative heat flux in the case of temperature distribution given by Eq. (2) and the open boundary of computational domain at $\tau_{zb} = 3.5$, and Fig. 4 shows the results in the case of temperature distribution given by Eq. (3) and the open boundary of computational domain at $\tau_{zb} = 3.0$. In the case simulated in Fig. 3, there is an isothermal zone with the optical thickness of $\Delta\tau_z = 0.5$ after the open boundary of computational domain, and in the case of simulated in Fig. 4, the optical thickness of the isothermal zone after the open boundary of computational domain is 1.0. As shown in Figs. 3 and 4, when the isothermal zone after the open bound-

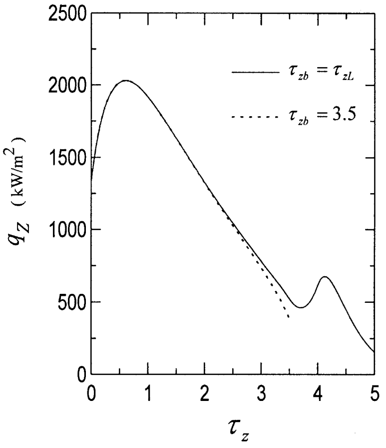

Fig. 3. The distribution of averaged axial net radiative heat flux in the case of temperature distribution given by Eq. (2) and the open boundary of computational domain locating at $\tau_{zb} = 3.5$.

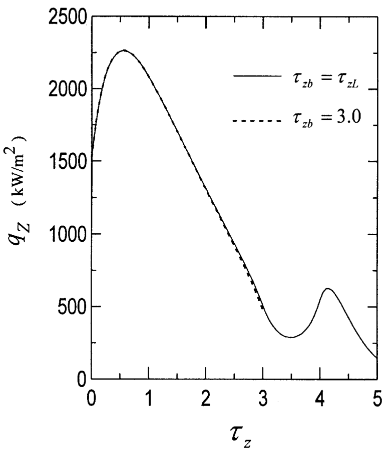

Fig. 4. The distribution of averaged axial net radiative heat flux in the case of temperature distribution given by Eq. (3) and the open boundary of computational domain locating at $\tau_{zb} = 3.0$.

ary of computational domain is long enough, its effective radiation is equated to that of a blackbody at local temperature, therefore, the errors resulting from taking the open boundary of computational domain as a black wall with local temperature is small, even if the open boundary of computational domain locates in high-temperature zone of 2000 K.

4. Conclusions

When the open boundary locates in the high-temperature zone and the temperature gradient is large, the error resulted from the open boundary to be treated as a black wall with local temperature may be very large and cannot be omitted. In order to get the reasonable numerical results, the open boundary needs to be set up far from the interesting zone more than 1.0 optical thickness if the open boundary of computational domain is treated as a black wall with local temperature. When the isothermal zone after the open boundary of computational domain is longer than 1.0 optical thickness, the errors resulting from taking the open boundary of computational domain as a black wall with local temperature are small, even if the open boundary of computational domain locates in high-temperature zone.

Acknowledgements

The supports of this work by Fok Ying Tung Education Foundation (no. 71053) and National Natural Science Foundation of China (no. 50176011) are gratefully acknowledged.

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